A Theoretical Investigation on Sound Transmission Loss through Multi-walled Plates with Air Space

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Authors’ contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJR2P/2019/46351

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Reviewers:

(1) Alice Elizabeth González, Universidad de la República, Uruguay.

(2) Fateh Mebarek Oudina, University of Skikda, Algeria.

Complete Peer review History: http://www.sdiarticle3.com/review-history/46351

Received 28 October 2018
Accepted 10 January 2019
Published 23 January 2019

ABSTRACT

In this study, an analytical model is proposed to investigate the sound transmission loss through multi-walled plates with air layers or decompression air layers, under the diffuse incidence field. Using the present approach, the influences of various parameters, such as the wall thickness, the decompressed air and the thickness of air space, on the sound transmission loss through are simulated and discussed in detail. It is seen that, due to the wave frequency of mass-air-mass resonance between double-walled glass plates, the sound transmission loss of the plates can be improved at low frequency range. The sound transmission loss tends to increase with decreasing air pressure because the sound is not transmitted through vacuum space. The design method can be used to investigate the effect of various geometric and material parameters on the sound transmission loss. The advantage of the simulation procedure is easily used for designing the layer structures with different parameter to improve the sound insulation effect.

Keywords: Transmission loss; multi-panel structures; acoustics; simulation; sound impedance.
1. INTRODUCTION

Improving the sound insulation performance of wall, window, ceilings and floors of general buildings, and building structures are very important aspects in living environment maintenance [1-3]. Many advanced composites, such as lightweight sandwich materials and multi-layered panels, have been widely applied in the fields including environment protection and energy conservation [4]. Important design parameters of walls or panels, such as material properties, the structures and the dimensions, affect largely on their sound insulation [5-8]. In the basic design stage of sound insulation products, the prediction of design parameters is very important before the products are developed and manufactured. Theoretical studies on the sound absorption and insulation are mainly based on the physical consideration of sound propagation. The mass law of transmission loss was a classical method, and generally adopted to predict the transmission loss through panels or walls, and used for frequencies between 2 \( f_{\text{frequency}} \) and 0.5 \( f_{\text{critical}} \) [9,10]. For the range at high frequency vibration, a number of simulation approaches on the sound insulation of plate structures were carried out using governing equations of motion, and there was good agreement between the theoretical and experimental results. Zhang et al. reported an effective medium model to predict the sound transmission loss through multi-layered panels [11]. The sound absorption properties of the thin plates could be investigated by explicit formulations derived from the effective medium method. For porous materials and perforated plates, the sound transmission loss of micro-perforated panel system was predicted theoretically using transfer matrix method based on lumped and distributed models [12-15]. In order to verify the theoretically predicted results, the experimental measurement of the sound absorption coefficient in the micro-perforated panel was performed using the impedance tube method and the reverberation room [12]. Takahashi and Tanaka [16] presented a method for analyzing acoustic coupling due to flexural vibration of perforated plates and plates of poroelastic materials. Moreover, some fundamental acoustic problems, such as the interactive effect of flexural vibration and plate permeability, were also investigated. Mahjoob et al. [17] investigated and discussed the acoustic insulation of multi-layered panels containing Newtonian fluids. A progressive impedance model was proposed to predict the sound insulation of the plate at normal incidence condition. The sound transmission loss through sandwich panels with damping layers were analyzed according to various structural and material properties [18-20]. Their results reported that the sandwich panel had better sound transmission characteristics than a homogeneous elastic panel of equivalent weight [18]. The sound insulation was largely affected by the damping materials and their properties for high frequency range [19]. The result shows that the skin damping alone did not decrease wave coincident peaks significantly. However, the damping in the core layer were more important than those in the skin layers, resulting in an increase of sound transmission loss with increasing damping. Natsuki [21] reported recently that the sound transmission loss through multi-layered panels can be improved based on the separated air space thickness or decreasing air pressure due to the influence of mass-airmass resonance. Recently, some researches on the sound transmission loss for cylindrical shells with multi-layers were investigated, based on Donnell’s shell theory [22] and the Biot’s theory [23]. Golizari and Jafari investigated the sound transmission loss through sandwich cylindrical shells according to Love’s theory and a simplified Biot’s theory, which described wave propagation in the porous material cores [24].

In this study, we suggest an analytical model and derive a formula for the diffuse field incidence of acoustic load through multi-layer panels. Based on the developed theoretical model [21], the influence of diffuse incidence, layer number and layer space on the sound transmission loss through multi-walled panels were investigated and analyzed in detail. The analytical results can be applied to design parameters to improve the performance characteristics of sound insulation.

2. THEORETICAL ANALYTICAL MODEL

2.1 Governing Equation

In the section, we developed the general formation for calculating the transmission loss of sound waves through multi-layered structures under the diffuse incidence field, and presented its solution. The progressive impedance model of multi-walled plates under scattering incidence is present to predict the sound transmission loss. Fig. 1 shows an analytical model of three-walled plates separated by air spaces or decompressed air layers. The dimension of plate is taken as...
infinite in the x-direction, and the thickness direction of plate is the z-axis. \( w_i \) denotes the transverse displacement of different plates with \( j = 1, 2, 3 \); \( p_i, p_r, \) and \( p_t \) are the incident, the reflected and the transmitted acoustic pressures, respectively. \( \rho_0 \) and \( c_0 \) are the air density and the sound speed at atmosphere, respectively. \( \rho' \) and \( c' \) are the decompressed density and the sound under the decompressed air, respectively.

The sound transmission is caused by the bending wave propagation in walls. According to Fig. 1, we obtain the equations of motion in terms of the transverse deflection, given by

\[
D_j w_i^{(4)} + \rho_j h_i \ddot{w}_i = \left[ p_{i\theta} + p_{r\theta} - p_{t\theta} - p_w \right]_{z=0} \tag{1}
\]

\[
D_2 w_2^{(4)} + \rho_2 h_2 \ddot{w}_2 = \left[ p_{2\theta} + p_{2r} - p_{2t} - p'_{w} \right]_{z=0} \tag{2}
\]

\[
D_3 w_3^{(4)} + \rho_3 h_3 \ddot{w}_3 = \left[ p_{3\theta} + p_{3r} - p_{3t} \right]_{z=0} \tag{3}
\]

and

\[
D_j = \frac{E_j h_j^3}{12(1-\nu_j^2)}, \quad j = 1, 2, 3 \tag{4}
\]

where \( \rho \) and \( h \) are the mass density and the thickness of plates, respectively. \( D, E \) and \( \nu \) are the flexural rigidity, the elastic modulus, and the Poisson’s ratio of plates, respectively.

### 2.2 Analytical Solution of Sound Transmission Loss

Considering harmonic wave vibration, the acoustic pressures of the incident, the reflected and the transmitted sound waves in Eqs. (1)-(3) can be expressed as

\[
p_i = P_i e^{i\omega t - ik(\sin \theta - z \cos \theta)}
\]

\[
p_r = P_r e^{i\omega t - ik(\sin \theta + z \cos \theta)}
\]

and, the transverse displacement of plates can be written as

\[
w_j = W_j e^{i\omega t - ik \sin \theta}, \quad j = 1, 2, 3
\]

where \( \theta \) is the angle of incidence, \( \omega \) is the angular frequency and \( k = \omega/c \) is the wave number of the sound velocity in air. \( W, P, P_r \) and \( P_t \) are the amplitudes of the vertical displacement of plates, the incident, reflected and transmitted acoustic pressures, respectively.

![Fig. 1. Schematic illustration showing a three-layered construction](image-url)

Substituting Eqs.(5)-(8) into Eqs.(1)-(3), we obtain

\[
D_j k^4 \sin^4 \theta W_i - \rho_j h_j \omega^2 W_i = P_{i\theta} + P_{r\theta} - P_{t\theta} - p_w
\]

\[
D_2 k^4 \sin^4 \theta W_2 - \rho_2 h_2 \omega^2 W_2 = P_{2\theta} + P_{2r} - P_{2t} - p'_w
\]

\[
D_3 k^4 \sin^4 \theta W_3 - \rho_3 h_3 \omega^2 W_3 = P_{3\theta} + P_{3r} - P_{3t}
\]
transmitted to incident sound, which is

Furthermore, substituting Eqs. (17) into Eq. (12), and Eq. (16) into Eq. (13), we obtain

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Furthermore, substituting Eqs. (17)-(18) into Eqs. (9)-(11), the motion equations can be expressed as a matrix form

where

From Eq. (14) and Eq. (19), the transmission coefficient is defined as the power ratio of the transmitted to incident sound, which is

\[
\tau(f, \theta) = \left| \frac{P_{tr}}{P_{ti}} \right|^2 = \frac{iZ\omega s_{21}s_{32}/\cos \theta}{s_{11}s_{22}s_{33} - s_{11}s_{22}s_{23} - s_{13}s_{12}s_{21}}\left(\frac{1 + e^{i2kd\cos \theta}}{1 + e^{i2kd\cos \theta}}\right)^2
\]
The transmission coefficient is a function of the incident angle $\theta$. For a diffuse sound field, the averaged form of the transmission coefficient $\bar{\tau}(f, \theta)$ can be obtained from integrating a semi-circular surface over all the angles. The averaged transmission coefficient over all angles of incidence $\theta$ is given by

$$\bar{\tau} = \frac{\int_{0}^{\theta_0} \tau(f, \theta) \sin \theta \cos \theta d\theta}{\int_{0}^{\theta_0} \sin \theta \cos \theta d\theta}$$

(24)

where the limiting $\theta_0$ is usually taken to be $78^\circ$, above which it is assumed that no sound is received. Thus, the sound transmission loss (STL) with random incidence is obtained as

$$STL = -10 \log_{10}|\bar{\tau}|$$

(25)

### 3. ANALYTICAL RESULTS AND DISCUSSION

In the study, the multi-walled plates are considered to be two- or three-layered glass panels with an air layer between them. The Young’s modulus of a glass plate is taken as 72 GPa, the density is 2200 kg/m$^3$, and the Poisson’s ratio is 0.3. The air density ($\rho$) and the sound velocity ($c$) can be calculated from the atmospheric pressure ($p$) and the absolute temperature ($T$), given by

$$\rho = \frac{p}{RT}, \quad c = \sqrt{\frac{\kappa}{\rho}}$$

(25)

where $R = 287 \text{ J/kg} \cdot \text{K}$ is the specific gas constant for air, and $\kappa = 1.403$ is the heat capacity ratio of the air.

Fig. 2 shows the sound transmission loss through two-walled glass plates subjected to the incident angles of 0 and 45 degrees, and the scattering diffuse angle. The simulated result shows that the sound transmission loss depends largely on the incidence angles. Comparing the result of scattering angle with oblique incidences, the sound transmission loss is the maximum for the case of normal incidence. The dips shown in Fig. 2 give the mass-air-mass resonance frequencies, called coincidence critical frequency, of glass panels separated by air space. The coincidence critical frequency is clearly observed to be large for an identical incidence, but the dips become weak for the scattering incidence. According to the present prediction, the critical frequency is about 900 Hz for the glass panels subjected to a normal incidence, and shifts to higher frequency with increasing incidence angle. The dips of the critical frequency for the scattering incidence are found to be lower than that for the identical incidence, and the sound insulation performance of the two-walled glass plates decreases.

In the following simulation, we considered general forms of the sound propagation and analyzed the sound transmission losses through panels based on scattering incidence of sound. The influence of layer numbers on the sound transmission loss are shown in Fig. 3 as a function of vibration frequency. The two- or three-walled panels are considered to have the same total thickness of 6 mm and the air space of 2 mm thick. The dip numbers of coincidence critical frequency increases with the increase of air layers because of interference vibration between adjacent plates. At low frequency less than 400 Hz (coincidence frequency), the multi-walled glass panel is observed to provide more high sound transmission loss, compared with single panel due to internal air space. A single glass panel has the largest sound insulation when the frequency is over 500 Hz. The influences of air space on the sound insulation of double-walled panels are shown in Fig. 4, when the air interval between the two plates are 2 mm, 5mm, and 10 mm, and the thickness of each glass panel is 3 mm. It can be seen that the coincidence critical frequency of the glass panels decreases with the increase of air space. The influence of the air space on the sound transmission loss is very small for high frequency, except close to the coincidence frequency value.

Fig. 5 shows the sound transmission loss of the double-walled panels with separated by air space of 2.0 mm. The total panels are 6.0 mm thick, in which the thickness ratio ($t_1 : t_2$) are (1, 1), (1, 1.2) and (1, 2), respectively. It is observed that the thickness ratio of the double-walled panel affects the sound transmission loss, especially for the coincidence frequency larger than 1000 Hz. The sound insulation of double-walled panels can be largely improved just introducing a little variation. This can be explained that the difference in plate thickness
cause more energy loss of sound pressure due to the mismatch in the resonance frequency between two plates.

Fig. 2. Effect of the incidence angle on the sound transmission loss through two layers of panels with 2 mm thick and separated by an air space of 2 mm.

Fig. 3. Sound transmission loss through different panels, having the same total thickness but different layer numbers

Fig. 4. Influences of different air space on the sound insulation of panels with 3 mm thick

Fig. 5. Sound transmission loss of double-walled panels with a different thickness ratio and an air space of 2 mm

Fig. 6 shows the influence of the decompressed pressure on the sound transmission loss of double-walled panels separated by air space of 2.0 mm. In the simulation, both layers between glass plates have the same pressure value, taken as the standard atmospheric pressure (1.0 atm), 1/5 atm and 1/10 atm with decompressed air. The sound insulation loss was precipitated for identical wall thickness of 3.0 mm (Fig. 6), and different wall thickness ratio (Fig. 7). The results suggest that both sound transmission loss and critical frequency are affected largely by decompressed air. The critical frequency decreases with decreasing air pressure, which results in improvement of the sound insulation ability in high frequency range. The sound transmission loss increases by about 5 dB for the vibration frequency over 3000 Hz, when the air pressure decreases to below 1/5 atm. According to mass law of transmission loss, this means that if the sound insulation performance will increase by 5 dB, the thickness of glass panel should be doubled. Therefore, the simulation suggests that decompressed air layer between the glass plates can effectively improve the performance of sound insulation.

For a different thickness ratio of 1:2, the influence of the decompressed pressure on the sound insulation is obtained from Fig. 7. It is observed that the effect of decompressed pressure on the sound insulation clearly increase for the double-walled panel for different wall thickness ratio. A comparison between simulation and experimental data based on literature are carried out. The measurements of...
the sound transmission loss are done on glass panels with (6-12-6) mm [25], indicating that there is good agreement in low vibration frequency region. The coincidence frequency of the panel with (6-12-6) mm is about 200 Hz.

In summary, the mass-air-mass resonance frequencies (coincidence critical frequency) can be moved out of interest frequency, by adjusting such as the decompressed air, the air space and the different thickness ratio of panels. Therefore, the sound transmission performance of multi-walled panels can be improved by shifting resonance frequencies.

4. CONCLUSIONS

A theoretical model and an exact solution procedure are proposed to predict the sound transmission loss of multi-walled structures with air layer. The motion equations of the multi-walled plate are obtained based on the bending vibration mode and acoustic pressure method, and the solution of the motion equation is derived. According to the proposed theoretical model and analysis, we investigate the influences of various parameters on the sound insulation loss of multi-walled panels. The sound insulation characteristics of the multi-walled panels is largely affected by air-space interval, plate thickness ratio and decompressed pressure. The result suggests that the sound transmission performance of panels with air space can be improved through design parameters optimization.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:
The peer review history for this paper can be accessed here:
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